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Transient Response of Two Fluid-Coupled Spherical Elastic Shells to an Incident Pressure Pulse

H. HUANG

Ocean Technology Division

December 23, 1977

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The transient response of a system of two concentric spherical elastic shells coupled by an ideal fluid and impinged by an incident plan pressure pulse is analyzed. The classical techniques of separation of variables and Laplace transforms are employed for solving the wave equations governing the fluid motions and the shell equations of motion. A scheme of iterative convolution was devised for the inversion of the Laplace transforms that facilitates the calculation of accurate transient solutions of the response of the shells. A sample calculation of shell responses was performed and results are compared to the case in which the outer shell is absent. This set of results demonstrates that a thin outer shell tends to be transparent to the incident pulse.

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TRANSIENT RESPONSE OF TWO FLUID-COUPLED SPHERICAL ELASTIC SHELLS TO AN INCIDENT PRESSURE PULSE

INTRODUCTION

An underwater weak shock wave sufficiently far away from its generating explosion source is often treated as an acoustic pulse. There is prolific literature on the studies of the transient interaction among such pressure waves and single elastic shells of simple shapes [1,2]. The results not only reveal many of the essential physical phenomena involved in the interaction problem but also are quite useful for the verification of approximation methods for predicting the underwater explosion response of submerged structures surrounded by an exterior fluid medium of infinite extent [2-5]. In the present endeavor, the transient response of a system of two fluid coupled concentric spherical elastic shells impinged by an external incident plane shock wave is analyzed. This purports to gain physical insight in the response as well as to provide a data base for the development of general numerical methods for predicting the underwater explosion response of fluid coupled shell systems such as the double hull section of a submarine. The problem with the spherical geometry permits the separation of variables in the wave equation governing the fluid motion and the shell equations of motion. The use of Laplace transforms then facilitates the calculation of satisfactory transient solutions of the response of the shells.

A paper appears in the Russian literature [6] independently dealing with exactly the same problem. The results obtained, however, only pertain to the point-symmetric and the translational motions of the shells and these two terms of the series solution are insufficient for the description of the complete shell response. It would also seem that the scheme of dual Volterra integral equations for the calculation of the inverse Laplace transform used in Ref. 6 is unnecessarily complex and numerically inefficient. It can be readily shown that only a single and simpler integral equation is needed if this scheme is used.

In this report, a simple, straightforward method is devised for the inverse Laplace transform, and the asymptotic behaviors of the shells are analytically discussed. An example calculation is also carried out using eight terms of the series solution for adequate convergence of shell displacements, velocities, and stresses. Time histories of the transient reponse of the interior shell are presented. This example demonstrates that a thin exterior shell tends to be transparent to the incident pulse.

Manuscript submitted December 16, 1977.

Description of the Problem

Figure 1 sketches the fluid-coupled spherical shell system and the incident plane pressure wave. The fluid surrounding the outer shell and that between the two shells are considered to be ideal compressible fluids in linear wave motions and can be characterized by their unperturbed mass densities and sound speeds, i.e., by (ρ^e, c^e) and (ρ, c), respectively. The shells are initially concentric. In this study, the strength of the incident wave is sufficiently weak such that the shell deflections are elastic and small and the deviation from the concentricity remains negligible for the time duration of interest. The mass densities, Young's moduli and Poisson's ratios of the outer and inner shells are (ρ_s^e, E^e, ν^e) and (ρ_s, E, ν), respectively. The middle surface radii and thicknesses of the outer and inner shells are (a^e, h^e) and (a, h), respectively. The Φ-coordinate of the spherical system (r, θ, Φ) is not shown in Fig. 1 since it is not needed due to the symmetry of the problem. The origin O coincides with the unperturbed center of the shells.

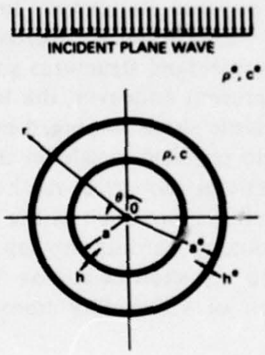

Fig. 1 – Geometry of the problem

The deflections of the inner shell in the r- and θ-direction, normalized with respect to the outer shell radius a^e, are denoted by w and u respectively and those of the outer shell by w^e and u^e respectively. The total pressure field exterior to the outer shell is denoted by $p^e(r, \theta, t)$ and that between the shells by $p(r, \theta, t)$ where t designates time. The following dimensionless parameters will be used in the mathematical formulation:

$$
\begin{aligned}
R &= r/a^e,\ T = c^e t/a^e,\ \zeta = a/a^e,\ \eta = c_r(1-\zeta),\\
c_r &= c^e/c,\ \rho_r = \rho/\rho^e,\ M = \rho^e a/(\rho_s h),\ M^e = \rho^e a^e/(\rho_s^e h^e),\\
\Pi &= p/[\rho^e (c^e)^2],\ \Pi^e = p^e/[\rho^e(c^e)^2],\ I = \frac{1}{12}(h/a)^2,\ I^e = \frac{1}{12}(h^e/a^e)^2,\\
C^2 &= E/[\rho_s(1-\nu)(c^e)^2],\ C_e^2 = E^e/[\rho_s^e(1-\nu^e)(c^e)^2],\\
\alpha_0 &= \alpha_0^e = \mu_0 = \mu_0^e = 0,\ \lambda_0 = 2C^2,\ \lambda_0^e = 2C_e^2,\\
\alpha_m &= (1+I)C^2[m(m+1)-(1-\nu)]/(1+\nu),\\
\alpha_m^e &= (1+I^e)C_e^e[m(m+1)-(1-\nu^e)]/(1+\nu^e),\\
\lambda_m &= C^2\{2+[1+(m^2+m+1)I][m(m+1)-(1-\nu)]/(1+\nu)\},\\
\lambda_m^e &= C_e^2\{2+[1+(m^2+m+1)I^e][m(m+1)-(1-\nu^e)]/(1+\nu^e)\},\\
\mu_m &= C^4(m^2+m-2)\{1-\nu+[m(m+1)-(1-\nu)]\times\\
&\quad [m(m+1)-(1+\nu)]I/(1-\nu)\}/(1-\nu)\\
\mu_m^e &= C_e^4(m^2+m-2)\{1-\nu^e+[m(m+1)-(1-\nu_e]\times\\
&\quad [m(m+1)-(1+\nu_e)]I^e/(1-\nu_e)\}/(1-\nu_e)\\
m &= 1, 2, 3, \ldots.
\end{aligned}
\qquad (1)
$$

Π^e and Π satisfy the wave equations

$$\nabla^2 \Pi^e = \frac{\partial^2 \Pi^e}{\partial T^2} \tag{2}$$

and

$$\nabla^2 \Pi = c_r^2 \frac{\partial^2 \Pi}{\partial T^2} \tag{3}$$

respectively, where ∇^2 is the Laplacian operator. The boundary conditions of the problem are that Π^e satisfies the radiation condition at far field and that

$$\frac{\partial \Pi}{\partial R} = \frac{\partial \Pi^e}{\partial R} = -\frac{\partial^2 w^e}{\partial T^2} \text{ at } R = 1 \tag{4}$$

and

$$\frac{\partial \Pi}{\partial R} = -\rho_r \frac{\partial^2 w}{\partial T^2} \text{ at } R = \zeta. \tag{5}$$

All quantities except the incident pressure field have quiescent initial conditions.

A Laplace transform pair is defined as

$$\overline{w}(\theta, s) = \int_0^\infty w(\theta, T) e^{-sT} dT$$

$$w(\theta, T) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \overline{w}(\theta, s) e^{sT} ds \tag{6}$$

where γ lies to the right of all singularities of $\overline{w}$ in the complex s-plane and $i = (-1)^{1/2}$.

Due to the spherical geometry of the problem, the solutions can be expanded in terms of series of Legendre polynomials as the following.

$$\Pi(R, \theta, T) = \sum_{m=0}^{\infty} \Pi_m(R, T) P_m(\cos\theta)$$

$$\Pi^e(R, \theta, T) = \sum_{m=0}^{\infty} \Pi_m^e(R, T) P_m(\cos\theta)$$

$$w(\theta, T) = \sum_{m=0}^{\infty} w_m(T) P_m(\cos\theta) \tag{7}$$

$$w^e(\theta, T) = \sum_{m=0}^{\infty} w_m^e(T) P_m(\cos\theta)$$

$$u(\theta, T) = \sum_{m=1}^{\infty} u_m(T) \frac{dP_m(\cos\theta)}{d\theta}$$

$$u^e(\theta, T) = \sum_{m=1}^{\infty} u_m^e(T) \frac{dP_m(\cos\theta)}{d\theta}$$

where P_m is the Legendre polynominal of the first kind and mth degree.

In the Laplace transform domain, the equations of motion of the elastic shells are [7]

$$\bar{w}_m = \frac{-M\zeta\Pi_m(\zeta, s)(\zeta^2 s^2 + \alpha_m)}{\zeta^4 s^4 + \lambda_m \zeta^2 s^2 + \mu_m}$$

$$\bar{w}_m^e = \frac{-M^e[\Pi_m^e(1, s) - \Pi_m(1, s)](s^2 + \alpha_m^e)}{s^4 + \lambda_m^e s^2 + \mu_m^e}$$

$$m = 0, 1, 2, \ldots \tag{8}$$

$$(\zeta^2 s^2 + \alpha_m)\bar{u}_m = \left[C^2 + \frac{I}{1 + I}\alpha_m\right]\bar{w}_m$$

$$(s^2 + \alpha_m^e)\bar{u}_m^e = \left[C_e^2 + \frac{I^e}{1 + I^e}\alpha_m^e\right]\bar{w}_m^e$$

$$m = 1, 2, 3, \ldots \tag{9}$$

It should be noted that the solution method developed here is applicable for any linear elastic theory for the spherical shells. The choice of the version in Eqs. (8) and (9) is for the comparison of results previously obtained in Ref. 7.

The total pressure field exterior to the outer shell Π^e consists of the pressure due to the incident wave and those due to scattering and radiation by the outer shell. An arbitrary incident plane pressure wave impinging the vertex of the outer shell $(R = 1, \theta = 0)$ at $T = 0$ can be expressed by the following series [7]

$$\Pi^i(R, \theta, s) = f(s)e^{-s}\sum_{m=0}^{\infty}(2m + 1)i_m(Rs)P_m(\cos\theta), \tag{10}$$

where $f(s)$ is the Laplace transform of the time characteristics of the incident wave and $i_m(Rs)$ is the abbreviated notation for the modified spherical Bessel function of the first kind $[\pi/(2Rs)]^{1/2}I_{m+1/2}(Rs)$ [8].

Solutions in the Laplace Transform Domain

It can be shown that the solutions to the system of Eqs. (2) through (10) are:

$$\bar{\Pi}_m^e(R, s) = (2m + 1)\frac{f(s)e^{-s}}{k_m'(s)}[i_m(Rs)k_m'(s) - k_m(Rs)i_m'(s)] - \frac{s\bar{w}_m^e k_m(Rs)}{k_m'(s)} \tag{11}$$

$$\begin{aligned}\bar{\Pi}_m(R, s) = {} & \frac{\rho_r s}{c_r[i_m'(c_r\zeta s)k_m'(c_r s) - i_m'(c_r s)k_m'(c_r\zeta s)]} \\ & \times \{[\bar{w}_m^e k_m'(c_r\zeta s) - \bar{w}_m k_m'(c_r s)]i_m(c_r Rs) \\ & - [\bar{w}_m^e i_m'(c_r\zeta s) - \bar{w}_m i_m'(c_r s)]k_m(c_r Rs)\}\end{aligned} \tag{12}$$

$$\bar{w}_m^e = -\frac{(2m+1)}{\Delta_m(s)} \frac{\pi e^{-s}}{2s^2} f(s) M^e (s^2 + \alpha_m^e) \{ (\zeta^4 s^4 + \lambda_m \zeta^2 s^2 + \mu_m)$$

$$\times [k_m'(c_r s) i_m'(c_r \zeta s) - i_m'(c_r \zeta s) k_m'(c_r s)] + \rho_r M \frac{\zeta s}{c_r} (\zeta^2 s^2 + \alpha_m)$$

$$\times [i_m'(c_r s) k_m(c_r \zeta s) - i_m(c_r \zeta s) k_m'(c_r s)] \} \tag{13}$$

$$\bar{w}_m = -\frac{(2m+1}{\Delta_m(s)} \frac{\pi e^{-s}}{2s^2} f(s) \frac{\pi}{2c_r^3 \zeta s} \rho_r M M^e (\zeta^2 s^2 + \alpha_m)(s^2 + \alpha_m^e). \tag{14}$$

In the above equations $k_m(s)$ is the abbreviated notation for the modified spherical Bessel function of the third kind, $[\pi/2s]^{1/2} K_{m+1/2}(s)$. The prime denotes differentiation of the Bessel functions with respect to their arguments, and

$$\Delta_m(s) = \Big\{ (\zeta^4 s^4 + \lambda_m \zeta^2 s^2 + \mu_m) [k_m'(c_r s) i_m'(c_r \zeta s) - k_m'(c_r \zeta s) i_m'(c_r s)]$$

$$+ \rho_r M \frac{\zeta s}{c_r} (\zeta^2 s^2 + \alpha_m) [i_m'(c_r s) k_m(c_r \zeta s) - k_m'(c_r s) i_m(c_r \zeta s)] \Big\}$$

$$\times [k_m'(s)(s^4 + \lambda_m^e s^2 + \mu_m^e) - M^e s k_m(s)(s^2 + \alpha_m^e)]$$

$$- \frac{\rho_r}{c_r} M^e s (s^2 + \alpha_m^e)(\zeta^4 s^4 + \lambda_m \zeta^2 s^2 + \mu_m) k_m'(s)$$

$$\times [k_m'(c_r \zeta s) i_m'(c_r s) - i_m'(c_r \zeta s) k_m(c_r s)]$$

$$+ \left\{ \frac{\rho_r}{c_r} \right\}^2 M M^e \zeta s^2 (s^2 + \alpha_m^e)(\zeta^2 s^2 + \alpha_m) k_m'(s)$$

$$\times [i_m'(c_r s) k_m(c_r \zeta s) - i_m'(c_r \zeta s) k_m(c_r s)] \tag{15}$$

With use of the Tauber's theorem of Laplace transforms [9], some of the asymptotic behaviors of the shell responses at late time can readily be revealed from Eqs. (13) and (14). Specifically, for the case where the incident wave Π^i is a unit step wave, i.e., $f(s) = 1/s$,

$$\underset{T \to \infty}{w_0^e(T)} = \frac{-2C^2 c_r^2 (1 - \zeta^3) M^e + 3\zeta^3 \rho_r M M^e}{4C^2 C_e^2 c_r^2 (1 - \zeta^3) + 6\rho_r M \zeta^3 C_e^2 + 6\rho_r M^e C^2} \tag{16}$$

and

$$\underset{T \to \infty}{w_0(T)} = \frac{-3\zeta \rho_r M M^3}{4C^2 C_e^2 c_r^2 (1 - \zeta^3) + 6\rho_r M \zeta^3 C_e^2 + 6\rho_r M^e C^2}. \tag{17}$$

These shell deflections occur long after the incident wave has engulfed the outer shell and can also be found by static analysis.

Again, for the unit step incidence case,

$$\underset{T \to \infty}{\dot{w}_1^e(T)} =$$

$$\frac{-3M^e[(1+2\zeta^2)\rho_r M + 6(1-\zeta^3)]}{6\rho_r M(1+2\zeta^3) + 6M^e[(1-\zeta^3) + (2+\zeta^3)\rho_r] + \rho_r M M^e [1 + 2\zeta^3 + 2\rho_r(1-\zeta^3)] + 36(1-\zeta^3)} \tag{18}$$

and

$$\dot{w}_1(T)\underset{T\to\infty}{=}\frac{-9\rho_r MM^e}{6\rho_r M(1+2\zeta^3)+6M^e[(1-\zeta^3)+(2+\zeta^3)\rho_r]+\rho_r MM^e[1+2\zeta^3+2\rho_r(1-\zeta^3)]+36(1-\zeta^3)} \tag{19}$$

where the dot denotes differentiation with respect to T. These are late time translational velocities of the shells in the direction of propagation of the incident wave. It can be seen that the only condition under which $\dot{w}_1 = \dot{w}_1^e$ at late time is $\rho_r M = 3$, i.e., when the inner shell is neutrally buoyant in the interior fluid. Otherwise, they are not equal to each other. Therefore, when interpreting the results of the present problem for large values of T, the relative positions of the shell must also be examined. If, at some time, the deviation from concentricity becomes excessive, the results of analysis thereafter are no longer physically meaningful.

Formulae (16) through (19) are convenient for parametric studies of the effects of various shell properties and fluid properties on the symmetric and translational responses of the shells, and they are also quite useful for providing asymptotic checks for the numerical calculations.

The Inversion of the Transformed Solutions

The spherical Bessel functions can be expressed in finite series of elementary functions [8]. If this property is used, the transformed solutions, e.g., Eq. (14), can be rearranged in the following form:

$$\overline{w}_m(s) = \frac{\Gamma_m(s)}{A_m(s)e^{\eta s} - B_m(s)e^{-\eta s}}, \tag{20}$$

where

$$\begin{aligned}
\Gamma_m(s) &= -2(-1)^{m+1}(2m+1)(\rho_r/c_r)f(s)MM^e(\zeta s)^{m+1}(c_r s)^{2m+2}(s^2+\alpha_m^e)(\zeta^2 s^2+\alpha_m)\\
A_m(s) &= \{X_m(s)X_m(-c_r s)(s^4+\alpha_m^e s^2+\mu_m^e) + M^e s^2(s^2+\alpha_m^e)[Y_m(s)X_m(-c_r s)\\
&\quad - \rho_r Y_m(-c_r s)X_m(s)]\}[X_m(c_r\zeta s)(\zeta^4 s^4+\lambda_m\zeta^2 s^2+\mu_m)\\
&\quad + \rho_r M\zeta^2 s^2(\zeta^2 s^2+\alpha_m)Y_m(c_r\zeta s)]\\
B_m(s) &= \{X_m(s)X_m(c_r s)(s^4+\lambda_m^e s^2+\mu_m^e) + M^e s^2(s^2+\alpha_m^e)[Y_m(s)X_m(c_r s)\\
&\quad - \rho_r Y_m(c_r s)X_m(s)]\}[X_m(-c_r\zeta s)(\zeta^4 s^4+\lambda_m\zeta^2 s^2+\mu_m)\\
&\quad + \rho_r M\zeta^2 s^2(\zeta^2 s^2+\alpha_m)Y_m(-c_r\zeta s)]\\
X_m(s) &= \sum_{k=0}^{m+1}(m+3/2,k)2^{-k}s^{m+1-k} - m\sum_{k=0}^{m}(m+1/2,k)2^{-k}s^{m-k}\\
Y_m(s) &= \sum_{k=0}^{m}(m+1/2,k)2^{-k}s^{m-k}\\
(m+1/2,k) &= \frac{(m+k)!}{k!(m-k)!}.
\end{aligned} \tag{21}$$

Both A_m and B_m are $(3m + 11)th$ order real polynomials of s and, for the plane incidence case, Γ_m is a $(3m + 6)th$ order real polynomial.

Equations of the form of Eq. (20) appear in many applied problems such as wave propagation in layered media, transmission lines, etc. The exponential factors in the denominator signify the reflections of waves, in the present case, to and from $R = \zeta$ and $R = 1$. The dimensionless time for a disturbance to traverse the distance $(1 - \zeta)$ is η. There is a variety of schemes for calculating the inverse Laplace transform of Eq. (20). One of the standardized techniques is the D'Alembert expansion [10] by which Eq. (20) can be rewritten as

$$\bar{w}_m(s) = \frac{\Gamma_m(s)e^{-\eta s}}{A_m(s)}\left\{1 + \frac{B_m}{A_m}e^{-2\eta s} + \left(\frac{B_m}{A_m}\right)^2 e^{-4\eta s} + \left(\frac{B_m}{A_m}\right)^3 e^{-6\eta s} + \cdots + \left(\frac{B_m}{A_m}\right)^n e^{-2n\eta s} + \cdots\right\}.$$

$$n = 1, 2, 3, \ldots \tag{22}$$

Since A_m is a real polynomial of s, its roots can be accurately computed by well-established numerical procedures and the inversion of every individual term on the right-hand side of Eq. (22) can then be obtained by the method of residue. The calculation of residues, however, would be rather clumsy for terms with A_m of high power. The following scheme utilizing the convolution theorem is devised to circumvent this. From Eq. (22),

$$w_m(T) = w_m^0(T - \eta)H(T - \eta) + w_m^1(T - 3\eta) + w_m^2(T - 5\eta)H(T - 5\eta) + \cdots + w_m^n[T - (2n + 1)\eta]H[\Gamma - (2n + 1)\eta] + \cdots \tag{23}$$

where H is the Heaviside step function and

$$w_m^1(T) = w_m^0(T) - \int_0^T G_m(T - \tau)w_m^0(\tau)d\tau,$$

$$w_m^2(T) = w_m^1(T) - \int_0^T G_m(T - \tau)w_m^1(\tau)d\tau,$$

$$\vdots$$

$$w_m^n(T) = w_m^{n-1}(T) - \int_0^T G_m(T - \tau)w_m^{n-1}(\tau)d\tau. \tag{24}$$

In Eq. (24), $w_m^0(T)$ and $G_m(T)$ are the inverse transforms of $\Gamma_m(s)/A_m(s)$ and $[A_m(s) - B_m(s)]/A_m(s)$ respectively. They are to be first accurately obtained by the method of residue. This is quite practical with the current computing technology for up to moderate values of m. For large m asymptotic expansions for the spherical Bessel functions can be used and A_m and B_m will assume simpler forms. For the calculations of shell responses in the present problem, large m terms are not needed. The successive convolutions required in Eq. (24) can be conveniently programmed and carried out in a modern digital computer. Since $w_m^0(T)$ and $G_m(T)$ are composed of terms formed by an exponential multiplied by a trigonometrical function, Trauboth's fast convolution integration algorithm [11], which only requiries about the same number of computation steps as for ordinary integration, is used here.

It can be seen from Eqs. (23) and (24) that each successive integration advances the solution time by 2η and the number of successive integrations required depends on T and η. The control of numerical accuracy lies in finding the zeros of A_m and the subsequent numerical integrations, and both are well established numerical techniques. Other quantities such as $w_m^e(T)$ can be found by the same procedure or calculated from their relationships with w_m.

Equation (20) can also be transformed into the following Volterra integral equation:

$$w_m(T) - w_m(T-\eta)H(T-\eta) + \int_0^T G_m(\tau)w_m(T-2\eta-\tau)H(T-2\eta-\tau)d\tau$$
$$= w_m^0(T-\eta)H(T-\eta). \tag{25}$$

On close examination, however, the solution of Eq. (25) requires repeating exactly the same convolutions as in Eq. (24). Therefore it is quite unnecessary to use the numerical techniques of integral equations, even if only one single integral equation is involved, for obtaining the inverse Laplace transform of Eq. (20). On the contrary, the use of two simultaneous Volterra integral equations for obtaining the inverse Laplace transforms of $\overline{w}_m$ and $\overline{w}_m^e$, as in Ref. 6 would have unduly increased the numerical difficulty and the computation effort.

Results and Discussions

Numerical results are obtained for a case in which both shells are made of steel and both fluids are water. The material properties and dimensions used are such that

$$c_r = \rho_r = 1, \quad C^2 = C_e^2 = 17.79133$$
$$M^e = 32.09875, \quad M = 6.41975$$
$$h^e/a^e = 1/250, \quad h/a = 1/50$$
$$\zeta = 0.8, \quad \eta = 0.2. \tag{26}$$

The incidence is a step wave with $f(s) = 1/s$. The transient responses of the shells are calculated for four transit times of the incident wavefront, i.e., $T = 0$ to 8. For this duration of time, it requires twenty successive convolution integrations in Eq. (24). The parabolic rule is used for the numerical integration in conjunction with Trauboth's fast convolution scheme. Since numerical roundoff and truncation errors accumulate at each successive integration, the integration intervals $\Delta\tau$ are kept sufficiently small to minimize these errors. The roots of $A_m(s)$, and from these, $w_m^0(T)$, and $G_m(T)$, are evaluated with high accuracy before starting the convolution integration. It is also found that, similar to the case of a single shell [7], eight terms ($m = 0 - 7$) in the series solution are sufficient for the representations of the shell deflections, velocities, and stresses. The integration intervals used are: $\Delta\tau = 0.01$ for w_0 and w_1, $\Delta\tau = 0.0025$ for w_2, and $\Delta\tau = 0.00125$ for the higher terms. If the solution must be carried out for shorter time duration, say up to $T = 4$, a much coarser $\Delta\tau$ will suffice.

In all subsequent figures, the present solutions are ploted in dotted lines and compared to the solutions obtained by the method of Ref. 7 and plotted in solid lines for the case in which the outer shell is absent.

Figure 2 shows the results for the w_0's and $\dot{w}_1$'s. Similar to the single-shell case, the present results approach their respective asymptotic values after about two transit times, and

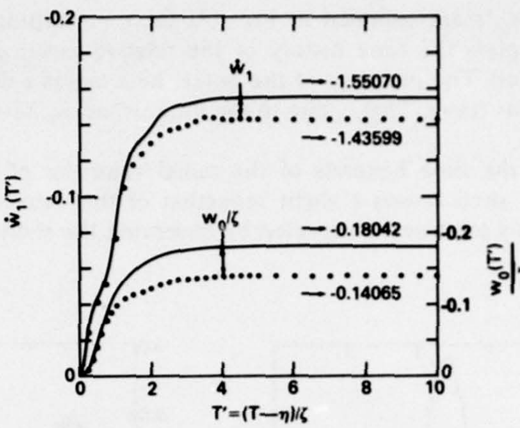

Fig. 2 – Time histories of w_0 and w_1

the numerical asymptotes of w_0 and $\dot{w}_1$ agree with formulae (17) and (19) respectively within three digits. These results are indicative of the accuracy of the present solution computation scheme. For this particular example, the outer shell is rather thin and its effect on the response of the inner shell is observable in the w_0, w_1, and w_2 terms. Its effect on the higher modes of the inner shell, i.e., w_m with $m > 2$, is quite undiscernible. Sample results for higher w_m's are exhibited in Fig. 3. It can be seen there that the present results for w_3 and w_4 coalesce with the respective results for the outer shell absent case. The same can be said about w_5, w_6, and w_7, and they are not replotted here.

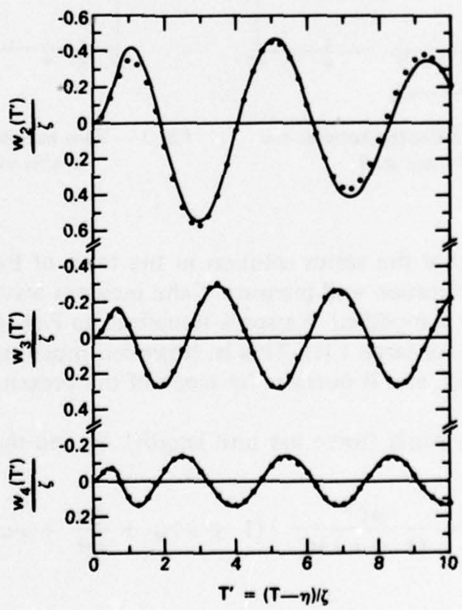

Fig. 3 – Time histories of w_2, w_3, and w_4

Eight terms of w_m's are summed in Eq. (7) for representing the total response of the inner shell. Figure 4 plots the time history of the relative radial deflection between the two apexes of the inner shell. The presence of the outer shell causes a downward shift of the curve representing the present result. This is due to the diminished w_0 term.

Figure 5 shows the time histories of the radial velocities of the same two apexes. The presence of the outer shell causes a slight reduction of the velocities with little alteration of their profiles. Again this can also be expected by observing the slight reduction of the $\dot{w}_1$ term.

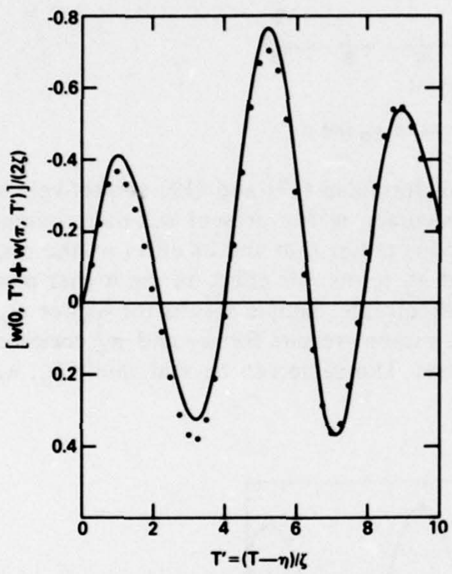

Fig. 4 – Relative radial deflection between the apexes of the inner shell

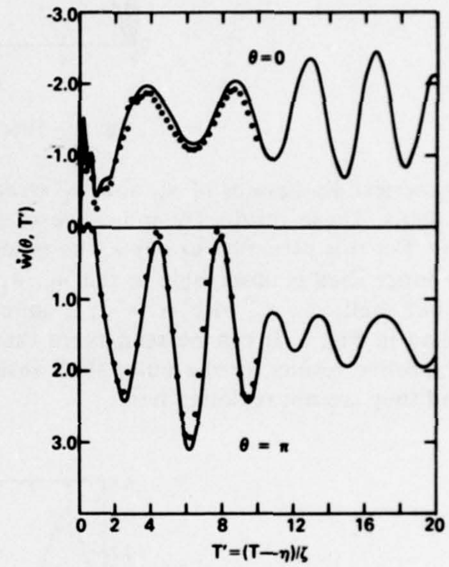

Fig. 5 – Time histories of radial velocities at the apexes of the inner shell

It is reiterated here that the series solution in the form of Eq. (7) is not effective for obtaining the early time acceleration and pressure if the incident wave has a steep front. The customary remedy is to apply a modified Watson's transform to Eqs. (11) through (14) and calculate the asymptotic results for large s [1]. This is, however, much more complex than the single shell case studied in Ref. 12, and is outside the scope of the present report.

The polar membrane stress (force per unit length) N_θ and the hoop membrane stress N_Φ are calculated by

$$N_\theta = \frac{aC^2}{(1+\nu)M\zeta}\left[(1+\nu)w + \frac{\partial u}{\partial \theta} + \nu u \cot\theta\right] \tag{27}$$

and

$$N_\Phi = \frac{aC^2}{(1+\nu)M\zeta}\left[(1+\nu)w + \nu\frac{\partial u}{\partial \theta} + u \cot\theta\right]. \tag{28}$$

The time histories of the polar and/or hoop stress at $\theta = \pi/2$ and at $\theta = \pi$ are plotted in Fig. 6 and 7 respectively. The shielding of the outer shell reduces the stresses slightly. The mean stresses also decrease in proportion to w_0. The maximum stresses occur after the incident wave has completely engulfed the outer shell and are about twice the corresponding static values.

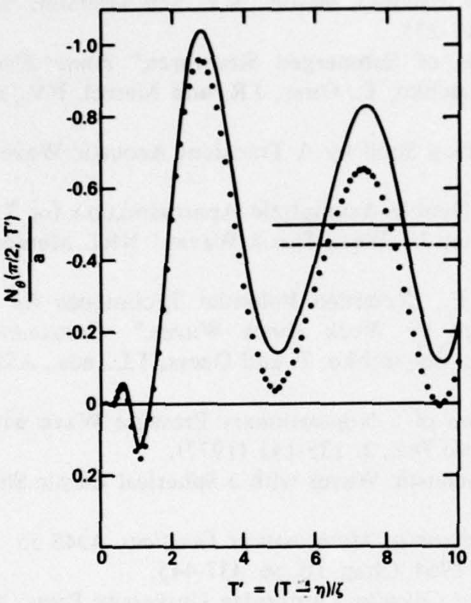

Fig. 6 – Time histories of the polar stress at $\theta = \pi/2$

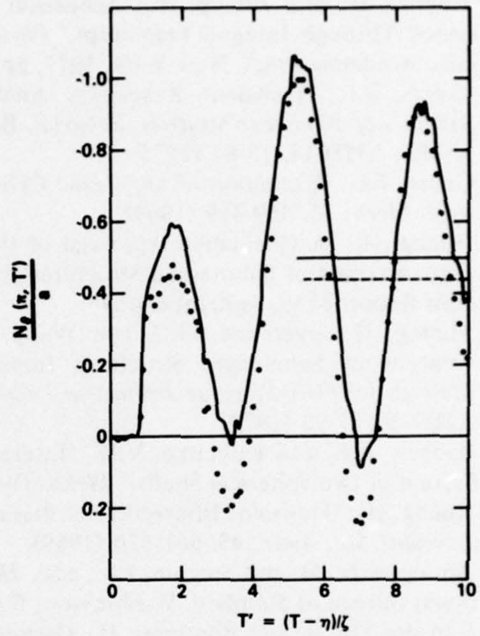

Fig. 7 – Time histories of the polar and hoop stresses at $\theta = \pi$

All results of the present example infer that a thin outer shell tends to be transparent to the incident pulse.

In conclusion, it can be said that an accurate solution method for this problem has been developed and can be used for parametric studies. Moreover, a set of numerical results has been meticulously obtained to serve as a data base for the development of numerical methods for the analysis of the transient response of double hulls of practical configurations. One immediate scheme is to apply the currently available Doubly Asymptotic Technique [2] in the outer fluid field and the fluid finite element technique for the entrained fluid.

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REFERENCES

1. Überall, H., and Huang, H., "Acoustical Response of Submerged Elastic Structures Obtained Through Integral Transform," *Physical Acoustics*, Mason, W.P., and Thurston, R.N., eds., Academic Press, New York, 1977, pp. 217-275.
2. Geers, T.L. "Transient Responses Analysis of Submerged Structures," *Finite Element Analysis of Nonlinear Structure Behavior*, Belytachko, T., Osias, J.R. and Marcel. P.V., eds., ASME, **AMD-14**, 15-84 (1975).
3. Geers, T.L., "Excitation of an Elastic Cylindrical Shell by A Transient Acoustic Wave," *J. Appl. Mech.*, **35**, 459-469 (1969).
4. Huang, H., "A Qualitative Appraisal of the Double Asymptotic Approximation for Transient Analysis of Submerged Structures Excited by Week Shock Waves," NRL Memorandum Report 3135, September 1975.
5. Huang, H., Everstine, G.C. and Wang, Y.F., "Retarded Potential Techniques for the Analysis of Submerged Structures Impinged by Week Shock Waves," *Computational Methods for Fluid-Structure Interaction Problems*, Belytschko, T. and Geers, T.L., eds., ASME, AMD-26, 83-93 (1977).
6. Babaev, A.E. and Kubenko, V.D., "Interaction of a Nonstationary Pressure Wave with a System of two Spherical Shells," *Mekh. Tverdogo Tela*, **2**. 135-141 (1977).
7. Huang, H., "Transient Interaction of Plane Acoustic Waves with a Spherical Elastic Shell," *J. Acoust. Soc. Amer.*, **45**, 661-670 (1969).
8. Abramowitz, M. and Stegum, I.A., eds., *Handbook of Mathematical Functions*, AMS 55, National Bureau of Standard, Washington, D.C., 1964, Chap. 10, pp. 437-445.
9. Van der Pol, B. and Bremmer, H., *Operational Calculus*, Cambridge University Press, New York, 1964, Chap. VII, pp. 122.
10. Ibid, Chap. IX, p. 189.
11. Trauboth, H.H., "Recursive Formulae for the Evaluation of the Convolution Integral," *J. Assoc. Comput. Mach.* **16**, (No. 1), 63-72 (1969).
12. Tang, S. and Yen, D., "Interaction of a Plane Acoustic Wave with an Elastic Spherical Shell," *J. Acoust. Soc. Amer.* **47**, 1325-1333 (1970).

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